



# CP and T Violation in Neutrino Oscillations <sup>1</sup>

Hisakazu Minakata <sup>a</sup>, Hiroshi Nunokawa <sup>b</sup> and Stephen Parke <sup>c</sup>

<sup>a</sup>*Department of Physics, Tokyo Metropolitan University  
1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan*

<sup>b</sup>*Instituto de Física Teórica, Universidade Estadual Paulista,  
Rua Pamplona 145, 01405-900 São Paulo, Brazil*

<sup>c</sup>*Theoretical Physics Department, Fermi National Accelerator Laboratory  
P.O.Box 500, Batavia, IL 60510, USA*

**Abstract.** In this short lecture, we discuss some basic phenomenological aspects of CP and T violation in neutrino oscillation. Using CP/T trajectory diagrams in the bi-probability space, we try to sketch out some essential features of the interplay between the effect of CP/T violating phase and that of the matter in neutrino oscillation.

## INTRODUCTION

There is now evidence for neutrino oscillations coming from the observation of atmospheric neutrinos [1], solar neutrinos [2], as well as neutrinos produced by accelerator [3]. In particular, recent evidence for the disappearance of  $\bar{\nu}_e$  coming from nuclear reactors reported by the KamLAND experiment [4] has finally established the so called large mixing angle (LMA) MSW [5] solution to the solar neutrino problem and has opened the door to explore the CP/T violating phase  $\delta$  in the Maki-Nakagawa-Sakata (MNS) [6] neutrino mixing matrix through neutrino oscillation [7]. In this talk we will discuss some basic aspects of CP and T violation in neutrino oscillation using bi-probability trajectories diagrams, which are quite useful for qualitative understanding of the subject.

## THREE NEUTRINO FLAVOR MIXING SCHEME

Let us consider the neutrino mixing among three flavor as

$$v_\alpha = \sum_{i=1}^3 U_{\alpha i} v_i \quad (1)$$

where  $v_\alpha (\alpha = e, \mu, \tau)$  and  $v_i (i = 1, 2, 3)$  are weak and mass eigenstates, respectively, and  $U$  is the MNS [6] neutrino mixing matrix. We will adopt the standard parametrization [8] of the MNS matrix as follows,

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

where  $c_{ij} \equiv \cos \theta_{ij}$ ,  $s_{ij} \equiv \sin \theta_{ij}$  and  $\delta$  is the CP violating phase.

---

<sup>1</sup> Invited talk given by H. Nunokawa at the 10 th Mexican School of Particles and Fields at Playa del Carmen, Mexico, Oct. 30 - Nov. 6, 2002

We already have significant amount of information about the MNS matrix as well as neutrino mass squared differences ( $\Delta m_{ij}^2 \equiv m_j^2 - m_i^2$ ) from various experiments. See, e.g., Ref. [9], for a recent review. Under the parametrization in Eq. (2), the atmospheric neutrino data indicate [1]

$$\sin^2 2\theta_{23} \simeq 0.88 - 1, \quad |\Delta m_{23}^2| \simeq |\Delta m_{13}^2| \simeq (1.5 - 4) \times 10^{-3} \text{eV}^2, \quad (3)$$

whereas the solar neutrino data combined with the KamLAND one indicate [10],

$$\tan^2 2\theta_{12} \simeq 0.25 - 0.85, \quad \Delta m_{12}^2 \simeq (4 - 20) \times 10^{-5} \text{eV}^2. \quad (4)$$

On the other hand, negative result of CHOOZ experiment impose constraint on  $\theta_{13}$  as [11],

$$\sin^2 2\theta_{13} \lesssim 0.1 \quad (5)$$

However, toward the complete understanding of the neutrino sector, there still remain following three questions to be answered, 1) what is the sign of  $\Delta m_{13}^2$ ?, normal ( $\Delta m_{13}^2 > 0$ ) or inverted ( $\Delta m_{13}^2 < 0$ )? 2) how small is the value of  $\theta_{13}$ ?, 3) what is the value of CP phase  $\delta$ ? Hereafter, we mainly focus on the effect of the CP phase  $\delta$  in neutrino oscillation, which can not be separately considered from that of the sign of  $\Delta m_{13}^2$  as well as the value of  $\theta_{13}$ , as we will see.

## CP AND T VIOLATION IN NEUTRINO OSCILLATION

Let us first consider the effect of CP and T violation in neutrino oscillation in vacuum [7]. If the oscillation probability of  $\nu_\beta \rightarrow \nu_\alpha$  is different from its CP conjugate process,  $\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha$ , or if

$$\Delta P_{\alpha\beta}^{CP} \equiv P(\nu_\beta \rightarrow \nu_\alpha) - P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \neq 0 \quad (\alpha, \beta = e, \mu, \tau, \alpha \neq \beta), \quad (6)$$

for a given neutrino energy ( $E$ ) and baseline ( $L$ ), then this implies CP violation. Similarly, if the oscillation probability of  $\nu_\beta \rightarrow \nu_\alpha$  is different from its T conjugate (time reversal) process,  $\nu_\alpha \rightarrow \nu_\beta$ , or

$$\Delta P_{\alpha\beta}^T \equiv P(\nu_\beta \rightarrow \nu_\alpha) - P(\nu_\alpha \rightarrow \nu_\beta) \neq 0 \quad (\alpha, \beta = e, \mu, \tau, \alpha \neq \beta), \quad (7)$$

then this implies T violation. If the CPT symmetry holds, which is the case for neutrino oscillation in vacuum, violation of T is equivalent to that of CP.

Using the parametrizaion in Eq. (2), one can explicitly show that in vacuum  $\Delta P_{\alpha\beta}^{CP}$  and  $\Delta P_{\alpha\beta}^T$  defined in Eqs. (6) and (7) are equal and given by,

$$\Delta P_{\alpha\beta}^{CP} = \Delta P_{\alpha\beta}^T = -16J_{\beta\alpha} \sin\left(\frac{\Delta m_{12}^2}{4E}L\right) \sin\left(\frac{\Delta m_{23}^2}{4E}L\right) \sin\left(\frac{\Delta m_{13}^2}{4E}L\right), \quad (8)$$

where

$$J_{\beta\alpha} \equiv \text{Im}[U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}] = \pm c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta, \quad (9)$$

with  $+$ ( $-$ ) sign is for cyclic (anti-cyclic) permutation of  $(\alpha, \beta) = (e, \mu), (\mu, \tau), (\tau, e)$ . Note that in order that the CP/T violation effect to be non-zero, all the angle must be non-zero and therefore, three flavor mixing is essential (no CP/T violation in two generation).

One can estimate the effect of CP/T violation in vacuum using the best fitted values of the mixing parameters obtained from solar and atmospheric neutrino data as,

$$\Delta P_{\alpha\beta}^{CP} = \Delta P_{\alpha\beta}^T \sim 2 \left[ \frac{\sin^2 2\theta_{12}}{0.83} \right]^{\frac{1}{2}} \left[ \frac{\Delta m_{12}^2}{7 \times 10^{-5} \text{ eV}^2} \right] \left[ \frac{\sin^2 2\theta_{13}}{0.05} \right]^{\frac{1}{2}} \sin \delta \quad [\%], \quad (10)$$

where we assumed that oscillation probability is measured at energy and baseline when  $\Delta m_{13}^2 L/4E$  takes  $\pi/2$  (oscillation maximal). The CP/T violation effect is expected to be a few percent provided that  $\theta_{13}$  is close to the CHOOZ limit, and can in principle be measurable by the proposed long-baseline neutrino oscillation experiments [12] in the near future. Let us note that if the solution to the solar neutrino problem were not LMA but some another one which requires much smaller  $\Delta m_{12}^2$  or  $\theta_{12}$ , then the measurement of CP/T violation effect would be much more difficult!

## CP AND T TRAJECTORIES IN BI-PROBABILITY SPACE

Let us now consider the effect of matter. In matter, measurement of CP violation can become more complicated because of the fact that oscillation probability for neutrinos and anti-neutrinos are in general different in matter even if  $\delta = 0$ . Indeed, the matter effect can either contaminate or enhance the effect of intrinsic CP violation effect coming from  $\delta$  [13]. For the case of T violation, the situation is different. If we can establish  $\Delta P_{\alpha\beta}^T \neq 0$  for  $\alpha \neq \beta$ , then this imply  $\delta \neq 0$  even in the presence of matter. This is because oscillation probability is invariant under time reversal even in the presence of matter. Similar to the case of CP violation, T violation effect can either be enhanced or suppressed in matter [14]. However, T violation measurement is experimentally more difficult to perform, because we need to make a non-muon neutrino beam!

Let us try to look into more about the interplay between the CP/T violation effect and matter effect. In order to have more transparent understanding of the subject, We will introduce the CP and T trajectory diagrams in the bi-probability space which were suggested and developed in Refs. [15, 16, 17]. From now on, we will focus on oscillation only for  $\nu_\mu \leftrightarrow \nu_e$  as well as  $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$  channels which are experimentally more feasible, because the production and detection of  $\nu_\tau/\bar{\nu}_\tau$  are much more difficult.

In vacuum one can show very easily that oscillation probabilities for neutrino and anti-neutrino channels take, without any approximation, the following forms,

$$\begin{aligned} P &\equiv P(\nu_\mu \rightarrow \nu_e) = A \cos \delta + B \sin \delta + C, \\ \bar{P} &\equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = A \cos \delta - B \sin \delta + C, \end{aligned} \quad (11)$$

where  $A$ ,  $B$  and  $C$  are some constant which depend on mixing parameters,  $\theta_{ij}$  and  $\Delta m_{ij}^2$ , as well as neutrino energy and baseline. Suppose that energy and baseline are fixed to some values. Then a given value of  $\delta$  (e.g.  $\delta = 0$ ) defines one point ( $P_0, \bar{P}_0$ ) in the  $P - \bar{P}$  plane. If we vary  $\delta$  from 0 to  $2\pi$  we can draw a closed trajectory, which is an ellipse, in

the  $P - \bar{P}$  plane. This is schematically illustrated in Fig. 1.

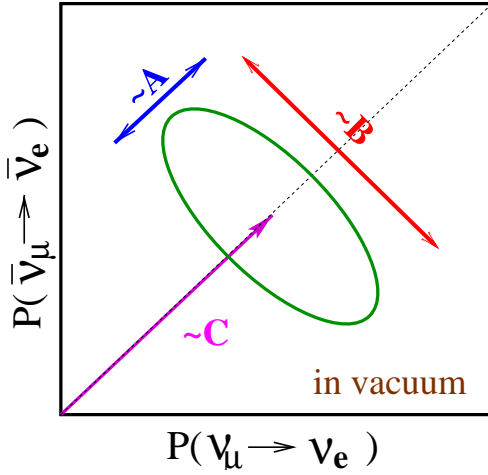


Fig. 1: Schematic illustration of the CP trajectory diagram without matter effect in the bi-probability  $P - \bar{P}$  plane.

trajectory, the two special points which sit at  $P = \bar{P}$  correspond to cases where  $\delta$  takes either 0 or  $\pi$ . Note that when  $\delta = 0$  or  $\pi$ , the mixing matrix become real and no difference between neutrino and anti-neutrino oscillation probability.

How the matter effect can change this picture? It has been noted in Ref. [18] that even in the matter with arbitrary density profile, the oscillation probability can be expressed in the same form as in vacuum in Eq. (11), without any approximation, as

$$P = \tilde{A} \cos \delta + \tilde{B} \sin \delta + \tilde{C}, \quad (12)$$

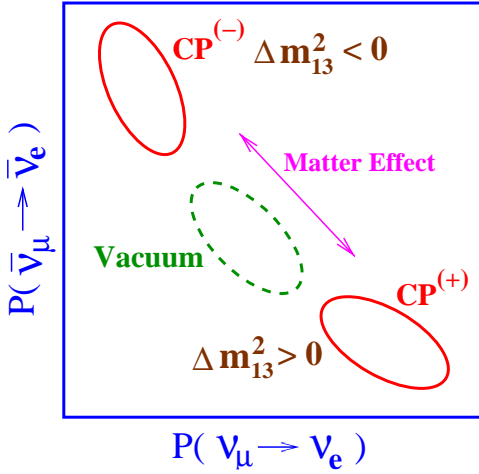


Fig. 2: Schematic illustration of how the trajectory in the  $P - \bar{P}$  plane is affected by the matter effect.

$\nu_\mu \rightarrow \nu_e$  ( $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ ) is enhanced whereas that for  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  ( $\nu_\mu \rightarrow \nu_e$ ) is suppressed so that they are sifted as in Fig. 2. The magnitude of shift is larger when the matter effect is larger, i.e., the baseline is larger. In fact, if the distance is short the two trajectories labeled as  $CP^{(+)}$  and  $CP^{(-)}$  can be overlapped (see Fig. 5 for an explicit example for such a case). We note that trajectories labeled as  $CP^{(+)}$  and  $CP^{(-)}$  are symmetric with respect to  $P = \bar{P}$  line, which we will explain why later.

There are several important features to be mentioned. First the constant  $C$ , which is proportional to  $\sin^2 \theta_{13}$ , determines how far the ellipse is located from the origin of the the  $P - \bar{P}$  plane. Second, the size of the ellipse, which is determined by the magnitudes of  $A$  and  $B$ , corresponds to the size of the effect of non-zero CP phase in oscillation probabilities. To be more precise, the size of the ellipse along the axis which is proportional to  $B$  characterize the size of direct CP violation effect which is proportional to  $P - \bar{P} \propto \sin \delta$  whereas that of the direction along the axis proportional to  $A$  characterize the effect of CP “conserving” term, which is proportional to  $\cos \delta$ . We note that the minor (if  $A > B$ ) or major (if  $A < B$ ) axis is always at 45 degree. On the

where  $\tilde{A}$ ,  $\tilde{B}$  and  $\tilde{C}$  are some constant which depend not only on mixing angle but also on the matter effect and are different for neutrinos and anti-neutrinos. This implies that the trajectory in matter is also elliptic but is shifted to two different directions, according to the sign of  $\Delta m^2_{13}$ , in the  $P - \bar{P}$  plane as illustrated in Fig. 2. What happens with matter effect is that the size of the trajectory does not change essentially but change its position in the  $P - \bar{P}$  plane due to some parallel shift plus rotation, as illustrated in Fig. 2. For positive (negative) value of  $\Delta m^2_{13}$ , due to the matter effect, probability for

Let us next consider the T violation. We

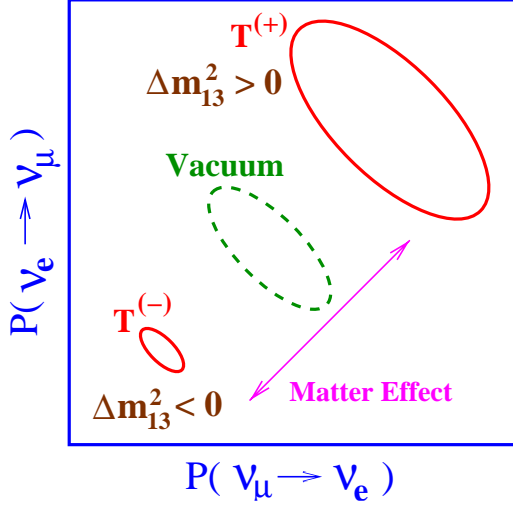


Fig. 3: Schematic illustration of the trajectory diagram in the bi-probability plane  $P - P^T$ .

## UNIFIED UNDERSTANDING OF CP AND T TRAJECTORIES

Now we will try to relate these two kind of different trajectories which represent CP and

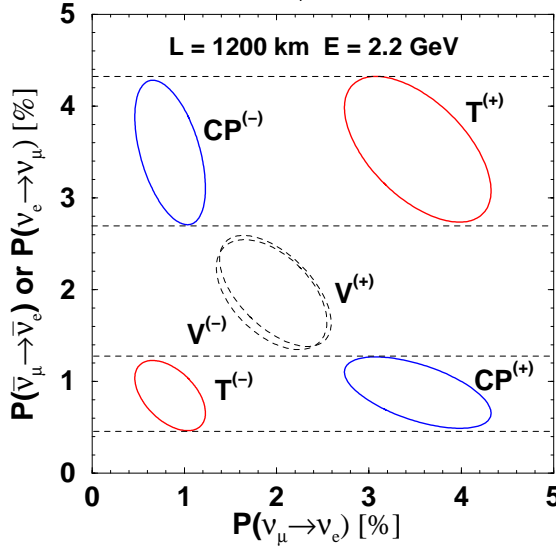


Fig. 4: Relation between CP and T trajectory diagrams.

can do exactly the same exercise as we did in Fig. 2 but in the bi-probability  $P - P^T$  plane where  $P^T \equiv P(\nu_e \rightarrow \nu_\mu)$  is probability for the T conjugate channel of the  $\nu_\mu \rightarrow \nu_e$  process. In contrast to the case of CP trajectory diagram, not only the position but also the size of the ellipse changes in matter as illustrated in Fig. 3. One can easily understand the qualitative behaviour of these trajectories by noting that due to the matter effect both  $P$  and  $P^T$  are enhanced (suppressed) for positive (negative)  $\Delta m^2_{13}$  [14]. The magnitude of the shift as well as the size change of the ellipse depend on the strength of the matter effect. Larger the matter effect (longer the baseline), larger the shift and the size change.

T violations in the presence of matter and give a unified picture [16]. In Fig. 4, we draw the CP and T trajectories in matter at the same time in  $P - \bar{P}$  or  $P - P^T$  plane together with that in vacuum (labeled as  $V^{(\pm)}$ ). For this plot, we set  $E = 2.2$  GeV,  $L = 1200$  km, and  $|\Delta m^2_{13}| = 3 \times 10^{-3}$  eV<sup>2</sup>,  $\sin^2 2\theta_{23} = 1.0$ ,  $\Delta m^2_{12} = +5 \times 10^{-5}$  eV<sup>2</sup>,  $\sin^2 2\theta_{12} = 0.80$ ,  $\sin^2 \theta_{13} = 0.05$  and the electron density  $Y_e \rho = 1.5$  g cm<sup>-3</sup>. The superscript  $(\pm)$  attached to the label CP/T/V indicate the sign of  $\Delta m^2_{13}$ . We note that two  $V^{(\pm)}$  trajectories are almost degenerated (We ignored the difference between  $V^{(+)}$  and  $V^{(-)}$  in the previous sections in Figs. 1-3 for simplicity). The vertical axis must be identified with  $\bar{P}$  and  $P^T$  for CP and T trajectories, respectively. There is a remarkable feature. Projections of  $CP^{(\mp)}$  and  $T^{(\pm)}$  trajectories to the vertical axis “almost” (but not exactly) coincide with each other. Let us try to explain why.

able feature. Projections of  $CP^{(\mp)}$  and  $T^{(\pm)}$  trajectories to the vertical axis “almost” (but not exactly) coincide with each other. Let us try to explain why.

In the presence of matter, one must consider the neutrino evolution equation given as,

$$i \frac{d}{dt} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \left[ U \text{diag}(0, \frac{\Delta m_{12}^2}{2E}, \frac{\Delta m_{13}^2}{2E}) U^\dagger + \text{diag}(a, 0, 0) \right] \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}, \quad (13)$$

where  $a = \sqrt{2} G_F N_e$  denotes the index of refraction for  $\nu_e$  in matter with  $G_F$  being the Fermi constant and  $N_e$  a constant electron number density in the earth. For anti-neutrinos, the same equation holds but with  $a \rightarrow -a$  and  $U \rightarrow U^*$ .

By taking the complex conjugate of this evolution equation, one can show that the equation for neutrino is identical to that of anti-neutrinos with the sign of  $\Delta m^2$ 's flipped, from which we can conclude that for arbitrary matter density profile,

$$P(\nu_\mu \rightarrow \nu_e; \Delta m_{13}^2, \Delta m_{12}^2, \delta, a) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e; -\Delta m_{13}^2, -\Delta m_{12}^2, \delta, a). \quad (14)$$

We call this CP-CP relation, which holds without any approximation.

On the other hand, by taking the time reversal ( $t \rightarrow -t$ ) of the evolution equation, one can show that the equation for neutrino which describe  $\nu_e \rightarrow \nu_\mu$  process is identical to that for its T conjugate process but for anti-neutrinos  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  with the signs of  $\Delta m^2$ 's and  $\delta$  flipped, from which we can conclude that without any approximation,

$$P(\nu_e \rightarrow \nu_\mu; \Delta m_{13}^2, \Delta m_{12}^2, \delta, a) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e; -\Delta m_{13}^2, -\Delta m_{12}^2, -\delta, a), \quad (15)$$

where we assumed that the matter density profile is symmetric about the mid-point between production and detection. Let us call this T-CP relation.

Now we would like to get similar relations but keeping the sign of  $\Delta m_{12}^2$  the same because we want to find some relations among  $\text{CP}^{(\pm)}$  and  $\text{T}^{(\pm)}$  trajectories for positive  $\Delta m_{12}^2$ , which is required by the solar neutrino data. By flipping the sign of  $\Delta m_{12}^2$  in the RHS of Eqs. (14) and (15), keeping only the order  $\Delta m_{12}^2 / \Delta m_{13}^2$ , one can get the following approximated CP-CP and T-CP relations [16],

$$P(\nu_\mu \rightarrow \nu_e; \Delta m_{13}^2, \Delta m_{12}^2, \delta, a) \simeq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e; -\Delta m_{13}^2, +\Delta m_{12}^2, \pi + \delta, a) \quad (16)$$

$$P(\nu_e \rightarrow \nu_\mu; \Delta m_{13}^2, \Delta m_{12}^2, \delta, a) \simeq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e; -\Delta m_{13}^2, +\Delta m_{12}^2, \pi - \delta, a). \quad (17)$$

Eq. (16) explains why the positions of  $\text{CP}^{(+)}$  and  $\text{CP}^{(-)}$  trajectories are approximately symmetric with respect to the line  $P = \bar{P}$  whereas Eq. (17) explains why the projections of  $\text{CP}^{(\mp)}$  and  $\text{T}^{(\pm)}$  to the vertical axis approximately coincide.

## PROBLEM OF PARAMETER DEGENERACY

Finally, let us mention briefly about the problem of parameter degeneracy [19, 15, 20]. Suppose that we can measure very precisely the oscillation probability  $P(\nu_\mu \rightarrow \nu_e)$  and its CP conjugate one  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  for a given energy and baseline. This give one point  $(P, \bar{P})$  in the  $P - \bar{P}$  plane. Assuming that we know all the mixing parameters, except for  $\theta_{13}$  and  $\delta$ , there is a situation where we can find four different CP trajectories which pass such a single point as illustrated in the left panel of Fig. 5. This implies that even if we can measure the probability very precisely, we can not distinguish such

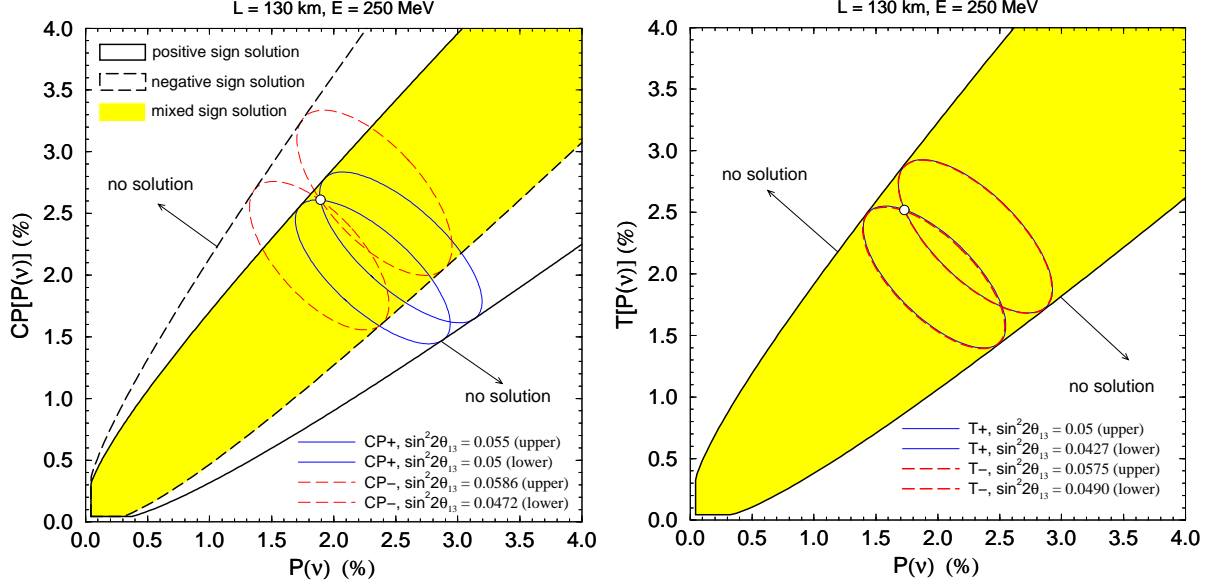


Fig. 5: Examples of the case where we have problem of 4-fold parameter degeneracy for CP violation (left) and T violation (right). Trajectories with solid (dashed) lines correspond to positive (negative)  $\Delta m_{13}^2$  and  $P(v) \equiv P(\nu_\mu \rightarrow \nu_e)$  and  $CP[P] \equiv \bar{P}$  and  $T[P] \equiv P^T$ . Same mixing parameters as in Fig. 4 except for  $\theta_{13}$ . Adopted from Ref. [17].

four different physical situations! This is the essence of the problem of parameter degeneracy. The shaded region in the plot indicate the region where we can not distinguish the sign of  $\Delta m_{13}^2$  by just measuring one set of  $(P, \bar{P})$ . In the case of T violation the problem become more serious as there are always four trajectories which pass such a single point in the  $P - P^T$  plane, corresponding to four physically different cases. For given  $P$  and  $\bar{P}$  ( $P^T$ ), possible set of solutions of  $(\theta_{13}, \delta)$  can be obtained analytically [17].

How to solve this problem? The possible answer is to perform experiments at two different energies and/or two different baselines [19, 21, 22], or to combine two different experiments [22, 23, 24, 25, 26]. Then the degeneracy will be lifted and in principle, we can uniquely determine the oscillation parameters, provided that we can measure oscillation probabilities accurately enough. See these references for detailed discussions.

## SUMMARY

We discussed some basic aspects of CP and T violation in neutrino oscillation in the presence of matter effect using bi-probability trajectory diagrams. These trajectory diagrams are quite useful for qualitative understanding of the subject as they show the effect of CP/T violation as well as matter effect at the same time in a single plot. We discussed the interplay among CP/T violation and matter effects and briefly mentioned about the problem of parameter degeneracy. We would like to conclude that we may have a good chance to observe CP and T violation in neutrino oscillation experiments in the next few years, provided that both  $\delta$  and  $\theta_{13}$  are not too small.

## ACKNOWLEDGMENTS

HN thanks organizers of the 10 th Mexican School on Particles and Fields for the invitation. HN also thanks Omar Miranda for the hospitality during his stay at CINVESTAV in Mexico and CONACyT for the financial support.

## REFERENCES

1. Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); Kamiokande Collaboration, H. S. Hirata *et al.*, Phys. Lett. B **205**, 416 (1988); *ibid.* **280**, 146 (1992); Y. Fukuda *et al.*, *ibid.* **335**, 237 (1994); IMB Collaboration, R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992); MACRO Collaboration, M. Ambrosio *et al.*, Phys. Lett. B **478**, 5 (2000); Soudan-2 Collaboration, W. W. M. Allison *et al.*, Phys. Lett. B **391**, 491 (1997); Phys. Lett. B **449**, 137 (1999).
2. Homestake Collaboration, B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1998); SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. C **60**, 055801 (1999); GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **447**, 127 (1999); GNO Collaboration, M. Altmann *et al.*, Phys. Lett. B **490**, 16 (2000); Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5651 (2001); *ibid.* **86**, 5656 (2001); S. Fukuda *et al.*, Phys. Lett. B **539**, 179 (2002); SNO Collaboration, Q. R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001); *ibid.* **89**, 011301 (2002); *ibid.* **89**, 011302 (2002).
3. K2K Collaboration, S. H. Ahn *et al.*, Phys. Lett. B **511**, 178 (2001); M. H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2001).
4. KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
5. S.P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).
6. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
7. The early references on leptonic CP/T violation include: N. Cabibbo, Phys. Lett. B **72**, 333 (1978); V. Barger, K. Whisnant and R. J. Phillips, Phys. Rev. Lett. **45**, 2084 (1980); S. Pakvasa, in *Proceedings of the XXth International Conference on High Energy Physics*, edited by L. Durand and L. G. Pondrom, AIP Conf. Proc. No. 68 (AIP, New York, 1981), Vol. 2, pp. 1164; S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. B **94**, 495 (1980); P. I. Krastev and S. T. Petcov, Phys. Lett. B **205**, 84 (1988); S. Toshev, Phys. Lett. B **226**, 335 (1989).
8. Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. **D66**, 010001 (2002).
9. M. C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. **75**, 345 (2003); S. Pakvasa and J. W. F. Valle, hep-ph/0301061.
10. V. Barger and D. Marfatia, Phys. Lett. B **555**, 144 (2003); G. L. Fogli *et al.*, Phys. Rev. D **67**, 073002 (2003); M. Maltoni, T. Schwetz and J. W. F. Valle, Phys. Rev. D **67**, 093003 (2003); A. Bandyopadhyay *et al.*, Phys. Lett. B **559**, 121 (2003); J. N. Bahcall, M. C. Gonzalez-Garcia and C. Peña-Garay, JHEP **02**, 009 (2003); H. Nunokawa, W. J. C. Teves and R. Zukanovich Funchal, Phys. Lett. B **562**, 28 (2003); P. Aliani *et al.*, hep-ph/0212212; A. B. Balantekin and H. Yuksel, J. Phys. G **29**, 665 (2003).
11. CHOOZ Collaboration, M. Apollonio *et al.* Phys. Lett. B **420**, 397 (1998); *ibid.* **466**, 415 (1999).
12. Y. Itow *et al.*, hep-ex/0106019; D. Ayres *et al.*, hep-ex/0210005; J. J. Gomez-Cadenas *et al.*, hep-ph/0105297.
13. See e.g., J. Arafune and J. Sato, Phys. Rev. D **55**, 1653 (1997); J. Arafune, M. Koike and J. Sato, Phys. Rev. D **56**, 3093 (1997) [Erratum *ibid.* D **60**, 119905 (1999)]; H. Minakata and H. Nunokawa, Phys. Rev. D **57**, 4403 (1998); Phys. Lett. B **413**, 369 (1997); Phys. Lett. B **495**, 369 (2000); O. Yasuda, Acta. Phys. Polon. B **30**, 3089 (1999).
14. S. Parke and T. J. Weiler, Phys. Lett. B **501**, 106 (2001).
15. H. Minakata and H. Nunokawa, JHEP **0110**, 001 (2001); Nucl. Phys. Proc. Suppl. **110**, 404 (2002).
16. H. Minakata, H. Nunokawa and S. Parke, Phys. Lett. B **537**, 249 (2002).
17. H. Minakata, H. Nunokawa and S. Parke, Phys. Rev. D **66**, 093012 (2002).
18. K. Kimura, A. Takamura and H. Yokomakura, Phys. Lett. **B537**, 86 (2002); *ibid.*, **B544**, 286 (2002).
19. J. Burguet-Castell *et al.*, Nucl. Phys. B **608**, 301 (2001).
20. V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D **65**, 073023 (2002).
21. V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D **66**, 053007 (2002).
22. J. Burguet-Castell *et al.*, Nucl. Phys. B **646**, 301 (2002).
23. A. Donini, D. Meloni and P. Migliozzi, Nucl. Phys. B **646**, 321 (2002).
24. V. Barger, D. Marfatia and K. Whisnant, Phys. Lett. B **560**, 75 (2003).
25. H. Minakata, O. Yasuda, H. Sugiyama, K. Inoue and F. Suekane, hep-ph/0211111.
26. P. Huber, M. Lindner and W. Winter, Nucl. Phys. B **654**, 3 (2003).